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$Z \to b\bar{b}$ and $Z \to c\bar{c}$ As Tests of the Standard Model

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We consider the decays $Z \to b\bar{b}$ and $Z \to c\bar{c}$ as tests of the Standard Model (SM). We find that $R_b = 0.2141$ and $R_c = 0.1719$. Comparing with recent experimental measurements, we find that $R_b^{\rm exp}$ is too high by 3.9σ and $R_c^{\rm exp}$ is too low by 2.4σ.

Recent precise experiments at LEP (CERN) have tested the Standard Model (SM) quite severely. In all cases the SM has performed well (Kniehl, 1994), with all experiments agreeing with the theoretical predictions. There have, however, been two notable exceptions. These are

$$R_b = \frac{\Gamma(Z - b\bar{b})}{\Gamma(Z \to \text{hadrons})} \tag{1}$$

and

$$R_c = \frac{\Gamma(Z - c\bar{c})}{\Gamma(Z \to \text{hadrons})} \tag{2}$$

The latest experimental value are

$$R_b^{\rm exp} = 0.2219(17) \tag{3}$$

and

$$R_c^{\rm exp} = 0.1543(74) \tag{4}$$

Using the SM values (Bardin *et al.*, 1995; Hagiwara, 1995)

$$R_b^{\rm th} = 0.2157 \tag{5}$$

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and

$$R_c^{\rm th} = 0.1724 \tag{6}$$

one finds discrepancies of 3.7σ and -2.5σ, for R_b and R_c, respectively. This has caused excitement in the high-energy physics community, with claims that the long-awaited breakdown of the SM has been found and that this is evidence of physics beyond the SM, such as supersymmetry (Wells *et al.*, 1994), etc. In this paper we will recalculate R_b and R_c, including previously omitted contributions, and find that

$$R_b^{\rm th} = 0.2141(11) \qquad (3.9\sigma) \tag{7}$$

and

$$R_c^{\rm th} = 0.1719(10) \qquad (-2.4\sigma) \tag{8}$$

where 3.9σ and -2.4σ are the deviations of the experimental values from the theoretical SM results in (7) and (8), respectively. As one can see, there is a large deviation, especially in the case of R_b. This represents a breakdown of the SM. For previous work on this subject, see Kniehl (1994), Bardin *et al.* (1995), Hagiwara (1995), Barbieri (1995), and Hagiwara *et al.* (1994, 1995). If we assign a reasonable error bar to (5) and (6), our results differ from the results in (5) and (6) by only 1.0σ and 0.3σ, respectively.

We now describe our calculations leading to the SM values in (7) and (8). All calculations are in the "on-shell scheme." We use $m_c = 1.35(5)$ GeV, $m_c(M_Z) = 725$ MeV, $m_b = 4.72(5)$ GeV, $m_b(M_Z) = 3.05$ GeV, $\sin^2\theta_w = 0.23143(28)$, $M_Z = 91.1884(22)$ GeV, $\alpha_s(M_Z) = 0.118(4)$, $\Gamma(Z \to \text{hadrons}) = 1.7407(59)$ GeV, and $M_t = 180(12)$ GeV, where the numbers in parentheses are the one-standard-deviation error bars. The parameters $m_b(M_Z)$ and $m_c(M_Z)$ are taken from Surguladze (1994) and the values of M_w and M_t are obtained from Renton (1995). The other parameters are taken from the Review of Particle Properties (1994).

We now use the recent mass-dependent QCD corrections of Surguladze (1994) to calculate the partial decay width of the Z boson:

$$\Gamma_{Z\to q_f\bar{q}_f} = \frac{G_F M_Z^3}{8\sqrt{2}\pi}\,\rho[(2I_f^{(3)} - 4e_f\sin^2\theta_w - \tau)^2 A + (2I_3^{(f)} - \tau)^2 B] \tag{9}$$

where the presence of $\tau \neq 0$ is due to the "GIM-violating contribution," which is present only in the $b\bar{b}$ case, where it represents the top correction to the $b\bar{b}$ vertex. The A and B are given in Surguladze (1994). The unknown term in equation (17) of Surguladze (1994) is estimated to be $-334(47)$ using our Padé method.

The relevant one-loop diagrams which contribute to both τ and ρ are shown in Fig. 3.1 of Wells *et al.* (1994). To second order, τ is given by

$$\tau = -2x\left[1 + x\left(9 - \frac{\pi^2}{3}\right)\right], \qquad r >> 1 \tag{10}$$

$$\tau = -2x\left[1 + \frac{x}{144}(311 + 24\pi^2 + 282 \ln r + 90 \ln^2 r)\right], \qquad r << 1 \tag{11}$$

where $r \equiv M_t^2/M_H^2$ and M_H is the (unknown) Higgs mass. See equation (15) for the definition of x.

From equations (12) and (13) and Barbieri *et al.* (1993) we estimate τ to be given by

$$\tau = -0.0069(10) \tag{12}$$

From equation (9) with $A = 1.039596$ and $B = 1.030985$ we find that, aside from the factor of ρ,

$$R_b = 0.2150(7) \tag{13}$$

To this must be added the virtual heavy quark contribution given by

$$\frac{\Delta\Gamma}{\Gamma_0} = \frac{-4x(a + b)}{a^2 + b^2} b\left\{R_0 + R_1\left[\frac{\alpha_s}{\pi}\right] + R_2\left[\frac{\alpha_s}{\pi}\right]^2\right\} \tag{14}$$

where $R_0 = 1$, $R_1 = (3 - \pi^2)/3$, and R_2 is estimated to be $R_2 = 5.2$ from our Padé method. Here x is given by

$$x = \frac{\sqrt{2}G_f M_t^2}{16\pi^2} = 0.00338(47) \tag{15}$$

and

$$\begin{aligned} a &= \tfrac{1}{3}\sin^2\theta_w - \tfrac{1}{4} = -0.1729 \\ b &= -0.25 \\ \Gamma_0 &= 0.3678 \text{ GeV} \end{aligned} \tag{16}$$

This leads to a decrease in R_b,

$$\Delta R_b = -0.0030(4) \tag{17}$$

Finally, we need to multiply by the factor ρ, which is given by equations (31)–(33) of Barbieri *et al.* (1993):

$$\rho = 1.0100(16) \tag{18}$$

Adding equations (13) and (17) and multiplying by ρ, we obtain our final result for R_b:

$$R_b = 0.2141(11) \tag{19}$$

This is somewhat lower than the previous estimate given in equation (5). If we neglect ρ and τ in equation (9), i.e., take $\rho = 1$ and $\tau = 0$, then we obtain

$$R_b = 0.2184 \tag{20}$$

Subtracting ΔR_b from (17), we obtain the final result

$$R_b = 0.2154 \tag{21}$$

This is very close to the previous SM result in equation (5).

In fact, if one looks at Hagiwara *et al.* (1994), equations (3.15)–(3.18), p. 571, one can see that $\rho = 1$ and $\tau = 0$ have been assumed. Table 5 in that paper shows that the result of their calculation for $m_t = 175$ GeV is $R_b = 0.2157$.

In the case of R_c the mass corrections are much smaller and we obtain $A = 1.038911$ and $B = 1.038411$. Using these values, we obtain our final result for R_c:

$$R_c = 0.1719(17) \tag{22}$$

This result is in good agreement with the previous SM result given in equation (6).

In conclusion, we have shown that there is a serious breakdown of the SM for R_b and R_c. If we do not assign a theoretical error, the deviations from experiment are 4.6σ and -2.4σ for R_b and R_c, respectively. If we include the theoretical error, the deviations are 3.9σ and -2.4σ, respectively. It turns out that the effect of our estimates from Padé on R_b and R_c is small. However, they serve the purpose of giving us assurance that unknown higher order corrections are negligible. In a recent paper Wells and Kane (1996) show that if one invokes supersymmetry (SUSY), the SUSY stop–chargino loop has the opposite sign of the t–W^+ loop and approximately cancels it if the stop and chargino are light enough. This would reduce the discrepancy in R_b to 2.4σ. Furthermore, when one includes this contribution and the increase in the Z width is used to determine α_s, one obtains a lower value of $\alpha_s(M_Z) = 0.112$, consistent with its determination in other ways. This strengthens one's confidence that both deviations are real, and also that the SUSY explanation is perhaps correct. If this is true, then the observation of SUSY particles, as well as the Higgs boson, is right around the corner.

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REFERENCES

Ball, P., Beneke, M., and Braun, P. (1995). *Nuclear Physics B452*, **563**.

Barbieri, R. (1995). The SM of the electroweak interaction, in *1994 European School of High-Energy Physics*, CERN 95-04, pp. 43–117.

Barbieri, R., Beccaria, M., Ciafaloni, P., Curci, G., and Viceré, A. (1993). *Nuclear Physics B*, **409**, 105.

Bardin, D., *et al.* (1995). Electroweak Working Group report, in *Reports of the Working Group on Precision Calculations for the Z Resonance*, D. Bardin, W. Hollik, and G. Passarino, eds., CERN 95-03.

Chankowski, P., and Pokorski, S. (1995). *Physics Letters B356*, **307**.

Erler, J., and Langacker, P. (1995). *Physical Review D*, **52**, 441.

Garcia, D., and Sola, J. (1995a). *Physics Letters B*, **354**, 335.

Garcia, D., and Sola, J. (1995b). *Physics Letters B*, **357**, 349.

Hagiwara, K. (1995). Implications of precision electroweak data, Presented at XVII International Symposium on Lepton and Photon Interactions at High Energies, August 1995, Beijing, China [fKEK-TH-461, HEP-PH/9512425].

Hagiwara, K., Haidt, D., Kim, S., and Matsumoto, S. (1994a). *Zeitschrift für Physik C*, **64**, 559.

Hagiwara, K., Haidt, D., Kim, S., and Matsumoto, S. (1994b). *Zeitschrift für Physik C*, **68**, 352E.

Kane, G., Stuart, R., and Wells, J. (1995). *Physics Letters B*, **354**, 350.

Kniehl, B. A. (1994). Status of higher-order corrections in the standard electroweak theory [KEK-TH-412].

Renton, P. B. (1995). Review of experimental results on precision tests of electroweak theories, Presented at XVII Symposium on Lepton and Photon Interactions at High Energies, August 1995, Beijing, China.

Review of Particle Properties (1994). *Physics Review D50*, 1173.

Shifman, M. (1995). *Modern Physics Letters A*, **10**, 605.

Surguladze, L. R. (1994). Quark mass corrections to the Z boson decay rates, University of Oregon OI-TS-554 [HEP-PH/9410409].

Wells, J. D., and Kane, G. L. (1996). *Physical Review Letters*, **76**, 869.

Wells, J. D., Kolda, C., and Kane, G. L. (1994). Implications of $\Gamma(Z \to bb)$ for supersymmetry searches and model building, University of Michigan, VM-TH-94-23.